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Maximally Tight Bell Inequalities Involving Lower Order Correlations

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Most of known multipartite Bell inequalities are for correlation functions for all subsystems. We propose a method of derivation of tight Bell inequalities involving also correlations of lower order. An example of a N qubit state, which violates such inequalities despite vanishing N-qubit correlations is given. We show how to derive a necessary condition for a state to violate such inequalities.

Entanglement might be seen as the most striking feature of quantum mechanics [1]. With perfect correlations, a feature of maximally entangled states Einstein, Podolsky, and Rosen [2] tried to argue that quantum theory could be supplemented with elements of reality respecting locality. This was disproved by Bell [3], who derived a theorem showing a discrepancy between all possible local realistic theories and entanglement. Since then Bell inequalities have been further generalized to more complicated experimental situations, and found applications in quantum cryptography, reduction of communication complexity, and other quantum information protocols. Series of new inequalities for an arbitrarily large collection of qubits, with two alternative observables per side were derived in Refs. [4, 5, 7–9]. Basing on similar ideas inequalities with more settings per site were derived in Refs. [10, 11].

Parallel to these developments, geometrical aspects of Bell inequalities were studied [12]. The set of all possible probabilities of experimental results, for experiments involving a finite number of settings, describable by local realistic theories forms a convex "Pitowsky" polytope in a vector space. The dimension of the space is determined by the number of particles, the dimensionality of the local observables, and by how many observables each observer is allowed to choose from. In case of N-partite this will be $D = 2^{\sum_{i=1}^{N} n_i}$ in case of inequalities presented here, $D = \prod_i (n_i + 1) - 1$, with n_i being the number of observables at disposal of the ith observer. The extreme points, or vertices, of this polytope are given by "deterministic models", that is apply to theoretical situations in which just one set of results is always obtained. For qubits, we have $K = 2^{\sum_{i=1}^{N} n_i}$. A tight Bell inequality is a one linked with a hyperplane spanned by at least D vertices of the polytope, all lying on the same facet, and thus defining it. All such vertices saturate the inequality. Here we would like to introduce a notion of maximally tight inequalities. We shall use this term whenever the associated hyperplane contains half of the vertices of the polytope. An example of such an inequality is the celebrated CHSH inequality [13], which takes value +2 for eight deterministic local realistic models, and −2 for the other eight. Basing on such concepts one can develop methods to derive maximally tight Bell inequalities for n qubits with three observables per site [14]. Some explicit forms of such inequalities were obtained in Ref. [15].

One can find multi-qubit states, which, despite being entangled, do not posses any correlations between all qubits [16]. In fact, they have non-zero correlations only between an even number of particles. Such states are now within the experimental reach, see for Dicke states refs. [17, 18] or for rotationally invariant states refs. [19]. Naturally, as there are no correlations between all qubits, such states cannot violate any of the aforementioned inequalities, as they involve correlations of the highest order.

Here we present an algorithm to derive tight Bell inequalities. These inequalities involve only some correlation functions for all N qubits, and mostly rely on lower order (in particular, $(N-1)$ qubit) correlations. We will demonstrate that they can be violated by states without any N-fold correlations, for an odd N.

Let us start with the simplest case, for which one can derive such inequalities, that is for three qubits. We assume that each observer can make one of two dichotomic measurements. The local realistic values, for the deterministic case, will be denoted as A^i_k, where $i = 1, 2, 3$ denotes the observer and $k = 1, 2$ is an index of the local observable. We put, as usual, the possible actual hidden values as $A^i_k = \pm 1$. The set of inequalities that we shall derive is based on the following algebraic identity:

$$\begin{aligned} &\tfrac{1}{8}(A^1_1 + A^1_2)(A^2_1 + A^2_2)(A^3_1 + A^3_2) \\ &\pm\tfrac{1}{4}(A^1_1 + A^1_2)(A^2_1 - A^2_2) \pm \tfrac{1}{4}(A^2_1 + A^2_2)(A^3_1 - A^3_2) \\ &\pm\tfrac{1}{4}(A^1_1 - A^1_2)(A^3_1 + A^3_2) \\ &\pm\tfrac{1}{8}(A^1_1 - A^1_2)(A^2_1 - A^2_2)(A^3_1 - A^3_2) = \pm 1, \end{aligned} \tag{1}$$

which we shall prove below, not by inspection, but using the general method that we want to present here.

The left hand side of (1) can be expressed as a scalar product of two tensors. The first one is built out of (deterministic) local realistic predictions, and reads

$$\hat{h}(\vec{A}) = (A^1_1, A^1_2, 1) \otimes (A^2_1, A^2_2, 1) \otimes (A^3_1, A^3_2, 1), \tag{2}$$

where $\vec{A} = (A^1_1, A^1_2, A^2_1, A^2_2, A^3_1, A^3_2)$. Note that there are $2^6 = 64$ tensors of such a kind. They are in a one-to-one way relation with the vertices of the Pitowsky convex polytope representing all possible local hidden variable representations of the problem, and thus we shall call

them simply *vertices*. The tensor component which is always equal to 1 plays here only a technical role.

Consider now another tensor built out of linear combinations of the following product tensors: $\hat{v}_{b_1b_2b_3} = v_{b_1} \otimes v_{b_2} \otimes v_{b_3}$, where $b_i = +1, -1, 0$, and $v_{\pm 1} = (1, \pm 1, 0)$ and $v_0 = (0, 0, 1)$. To simplify notation we shall denote the first two vectors as $v_{\pm}$, an use a similar convention for the tensors $\hat{v}_{b_1b_2b_3}$. Whenever we restrict the values of b_i's to ± 1, or $\pm$, we shall denote them as a_i. The vectors form a unnormalized basis in R^3. Next we construct a specific linear combination of $\hat{v}_{b_1b_2b_3}$ which has the property that its scalar product with any vertex has the same modulus, in our case 1, and additionally only *one* term of the linear combination has a non-zero scalar product with any of vertices. To this end, one must multiply any $\hat{v}_{a_1a_2a_3}$ by $\frac{1}{8}$. Next we observe that such tensors have $8 = 2^3$ vertices with which they are non-orthogonal. Those $\hat{v}$'s with just one $b_i = 0$ we multiply by $\frac{1}{4}$. Each of them has two times more vertices, than the tensors of the earlier class, which are not orthogonal. And so on. Of course, $\hat{v}_{000}$ cannot enter a non-trivial construction of such a kind, as they are non-orthogonal with respect to all vertices.

Our final task is to find a set of $\hat{v}$'s having the required property. We can start with and arbitrary one of the kind $\hat{v}_{\pm\pm\pm}$, and if we continue only with such ones, we shall end up with a WWWŻB inequality. To get anything new we have to use at least one $\hat{v}$ with at least one 0 index. So let us start with $\hat{v}_{+++}$, and in the second step add $\hat{v}_{+-0}$, as they certainly do not share any non-orthogonal vertex (generally $\hat{v}_{b_1b_2b_3}$ and $\hat{v}_{b_1'b_2'b_3'}$ do not share a non-orthogonal vertex, if they have a pair of indices satisfying $0 \neq b_k \neq b_k' \neq 0$). A similar situation happens if we add in the second step $\hat{v}_{0+-}$ or $\hat{v}_{-0+}$. Next notice that all of them do not share non-orthogonal vertices, so the whole set is in. Finally we see that we could have started with $\hat{v}_{---}$ to get the same triad with zeros, and that this vector does not share any non-orthogonal $\hat{h}$ with $\hat{v}_{+++}$. As there are only 64 vertices and $2 \times 8 + 3 \times 16 = 64$, we have built a linear combination of the required properties, which reads

$$\pm \frac{1}{8}(\hat{v}_{+++} \pm \hat{v}_{---}) \pm \frac{1}{4}(\hat{v}_{+-0} \pm \hat{v}_{-0+} \pm \hat{v}_{0+-}). \quad (3)$$

Obviously a "sum of products of components" scalar product of (3) and (2) gives (1).

Note important facts. The coefficients give us fraction of vertices of the Pitowsky polytope with which the given member of the combination is non-orthogonal (e.g., $\hat{v}_{+++}$ is non-orthogonal to $\frac{1}{8}$ of the vertices). Note further, that as the moduli of the coefficients add up to 1 by putting $\hat{v}'(b_1b_2b_3) = \pm\hat{v}(b_1b_2b_3)$, where $\pm$ now stands for the effective sign in the combination before the given tensor, we have a *convex* combination of the form

$$\frac{1}{8}(\hat{v}'_{+++} + \hat{v}'_{---}) + \frac{1}{4}(\hat{v}'_{+-0} + \hat{v}'_{-0+} + \hat{v}'_{0+-}).$$

After the usual averaging we get a set of 2^5 Bell inequalities (inequalities of this kind were found in ref. [20] using a completely different method, not allowing for easy generalizations)

$$\begin{aligned} &\pm\langle \tfrac{1}{8}(A_1^1 + A_2^1)(A_1^2 + A_2^2)(A_1^3 + A_2^3) \\ &\pm \tfrac{1}{4}(A_1^1 + A_2^1)(A_1^2 - A_2^2) \pm \tfrac{1}{4}(A_1^2 + A_2^2)(A_1^3 - A_2^3) \\ &\pm \tfrac{1}{4}(A_1^1 - A_2^1)(A_1^3 + A_2^3) \\ &\pm \tfrac{1}{8}(A_1^1 - A_2^1)(A_1^2 - A_2^2)(A_1^3 - A_2^3)\rangle \leq 1, \end{aligned} \quad (4)$$

which just as in the case of WWWŻB ones can be wrapped up to just one (this feature was not noticed in [20])

$$\begin{aligned} &|\langle \tfrac{1}{8}(A_1^1 + A_2^1)(A_1^2 + A_2^2)(A_1^3 + A_2^3)\rangle| \\ &+|\langle \tfrac{1}{4}(A_1^1 + A_2^1)(A_1^2 - A_2^2)\rangle| + |\langle \tfrac{1}{4}(A_1^2 + A_2^2)(A_1^3 - A_2^3)\rangle| \\ &+|\langle \tfrac{1}{4}(A_1^1 - A_2^1)(A_1^3 + A_2^3)\rangle| \\ &+|\langle \tfrac{1}{8}(A_1^1 - A_2^1)(A_1^2 - A_2^2)(A_1^3 - A_2^3)\rangle| \leq 1, \end{aligned} \quad (5)$$

Unfortunately ineq. (5) is not violated by any state. Still by extending the above method to more qubits we get "relevant" tight inequalities. Note further that by a consistent sign swap $\pm$ to $\mp$ for one particle, or more of them, in all terms of (5), we get other inequalities.

Let us now describe a general method for the N qubit case. We define $Z(N, k)$ as a certain set of N-fold tensor products of v_+, v_- and v_0 involving at most k v_0's, We shall again construct an algebraic identity, which can be interpreted as scalar product of a tensor which is a linear combination (interpretable as a convex one) of the elements of $Z(N, k)$, with another tensor, built out of local realistic values, of the form $\hat{h}(\vec{A}) = \otimes_{i=1}^N (A_1^i, A_2^i, 1)$, where now $\vec{A} = (A_1^1, A_2^1, ..., A_1^N, A_2^N)$. Obviously, the latter tensor is in a one-to-one relation with a vertex of the Pitowsky polytope of the problem. We shall call it *vertex*.

Although this is not necessary, we focus on $Z(N, k)$'s which are invariant with respect of a cyclic permutation of the indices of the tensors. We shall call such set as having a cyclic permutation symmetry (CPS).

The set $Z(N, k)$ is constructed in an iterative way. We can start to build $Z(N, k)$ with an initial set of tensors of the form $\hat{v}_{a_1a_2...} = v_{a_1} \otimes v_{a_2} ...$, hereafter denoted as $v_{\pm\pm...}$, each of which has a CPS. We extend the definitions given in the example to N particles, thus as before $a_i = \pm 1$, etc. For prime values of N, there are only two such vectors $v_{++...+}$ and $v_{--...-}$. Alternatively we could choose one of the proper subsets with CPS. In next steps, we supplement $Z(N, k)$ with specially chosen tensors $\hat{v}_{b_1...b_N}$ with at least one and at most k of b_i's equal 0. Before we proceed further we notice the following: vertices can be grouped into classes of equivalence. The following property the scalar products of a tensor $\hat{v}_{a_1a_2,...a_N}$ with the vertices define such classes: there are groups of 2^N vertices which have a non zero scalar products with it, equal to $\pm 2^N$. All other vertices are orthogonal to it. Such a class will be denoted by $h(\vec{a})$, where $\vec{a} = (a_1, ..a_N)$.

We repeat the following steps:

1. We make a table of scalar products between the vertices and vectors belonging to the current set $Z(N,k)$.

2. Out of all groups of vertices $h(\vec{a})$ orthogonal to all $\hat{v}$ in the current $Z(N,k)$ we choose 2^k such, which are different from each other at not more than k fixed positions (in the earlier example this would have been e.g. $h(+-+)$ and $h(+--)$). We take associated with them $\hat{v}_{a_1 a_2,\ldots a_N}$'s and replace v_+'s and v_-'s with v_0's at these chosen positions (in the example, we would obtain v_{+-0}).

3. We add to $Z(N,k)$ all cyclic permutations of the tensor obtained in the last step (involving v_0's).

This procedure is repeated until $Z(N,k)$ is complete, i. e., there are no vertices orthogonal to all elements of $Z(N,k)$. The structure of $Z(N,k)$ must be such that each vertex has a non-zero scalar product with exactly one tensor $\hat{v} \in Z(N,k)$. Only then it generates a maximally tight inequality. It may also happen that there are not enough vectors to complete step 2. This can happen, for example, while constructing $Z(7,3)$. One could then temporarily decrease k and continue the protocol, or simply leave $Z(N,k)$ incomplete, at the price of not getting a maximally tight inequality .

Finally we build a tensor:

$$\hat{Z}(Z(N,k)) = \sum_{Z(N,k)} S(\vec{b}) 2^{-B(\vec{b})} \hat{v}(\vec{b}), \tag{6}$$

where $B(\vec{b}) = \sum_{i=1}^{N} |b_i|$ and $S(\vec{b})$ is any sign function of the indices, that is of two values ± 1. Note that the factor $2^{-B(\vec{b})}$ guarantees that the term containing $\hat{v}(\vec{b})$ contributes to the scalar product with any vertex ± 1 or 0. The properties of the set $Z(N,k)$, provided it is complete, in turn guarantee that for any vertex $\hat{h}(\vec{A})$

$$\hat{Z}(Z(N,k)) \cdot \hat{h}(\vec{A}) = \pm 1, \tag{7}$$

where the scalar product, denoted a $\cdot$, is a "sum of products of components" one. This after averaging over any possible probabilistic distributions of vertices, gives a set of 2^z inequalities, where z is the number of elements in $Z(N,k)$ (this reflects the number of possible relative signs between the elements of the tensor $\hat{Z}$). One can wrap them into a single inequality using exactly the same method as in the case of the example for $N = 3$ given earlier.

Some remarks on the construction. The first observation is that, since the protocol does not determine which vectors will be chosen in each step, but rather leaves it up to us, there might exist different sets $Z(N,k)$ for the same values of N and k leaving to different inequalities.

The second point is about values of k. Certainly, we are interested in odd k's and odd N's. There are states which have correlations of even orders, but with no of odd ones. States with the opposite property are unknown (probably impossible). One must have $k < \frac{N}{2}$. This is because otherwise, there could be some vertices, which have a non-zero scalar product with more than one vector belonging to $Z(N,k)$. Too see this, e. g., take a pair of v_{+00} and v_{00+} of the $N = 3$ example. In such cases the construction cannot work.

Let us now present some specific sets $Z(N,k)$ and discuss their violation (CP denotes the set of all cyclic permutations, e.g., $CP(v_{+-0}) = \{v_{+-0}, v_{0+-}, v_{-0+}\}$). As it has been said above the interesting inequalities are those for odd N.

For five qubits we have found

$$Z(5,1) = \{v_{+++++}, v_{-----}\} \cup CP(v_{+++-0}) \cup CP(v_{-++-0}) \cup CP(v_{-+--0}), \tag{8}$$

for seven

$$\begin{aligned} Z(7,1) = \{v_{+++++++}, v_{-------}\} &\cup CP(v_{+++++-0}) \cup CP(v_{+++---0}) \cup CP(v_{-+--++0}) \cup CP(v_{-----+0}) \\ &\cup CP(v_{+-+-++0}) \cup CP(v_{++-++-0}) \cup CP(v_{--+-++0}) \cup CP(v_{+-+---0}) \cup CP(v_{+-++-+0}), \end{aligned} \tag{9}$$

and for nine, which is a non-prime number

$$
\begin{aligned}
Z(9,1) = & \{v_{+++++++++}, v_{---------}, v_{+--+--+--}, v_{-+--+--+-}, \\
& v_{--+--+--+}, v_{++-++-++-}, v_{+-++-++-+}, v_{-++-++-++}\} \\
\cup\ & CP(v_{+++++++-0}) \cup CP(v_{-++++++-0}) \cup CP(v_{-+++++-+0}) \cup CP(v_{-++++-++0}) \\
\cup\ & CP(v_{-+++++--0}) \cup CP(v_{-++++-+-0}) \cup CP(v_{-++++--+0}) \cup CP(v_{-++++---0}) \\
\cup\ & CP(v_{-+++-++-0}) \cup CP(v_{-+++-+-+0}) \cup CP(v_{-+++--+-0}) \cup CP(v_{-+++-+--0}) \\
\cup\ & CP(v_{-+++--++0}) \cup CP(v_{-+++----0}) \cup CP(v_{-+++---+0}) \cup CP(v_{-++-++--0}) \\
\cup\ & CP(v_{-++-+-+-0}) \cup CP(v_{-++-+---0}) \cup CP(v_{-++--++-0}) \cup CP(v_{-++-----0}) \\
\cup\ & CP(v_{-++---+-0}) \cup CP(v_{-++-+--+0}) \cup CP(v_{--++--+-0}) \cup CP(v_{-+-+--+-0}) \\
\cup\ & CP(v_{-+-+----0}) \cup CP(v_{--+--+-+0}) \cup CP(v_{----+--+0}) \cup CP(v_{---+----0}),
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
Z(9,3) = & \{v_{+++++++++}, v_{---------}, v_{+--+--+--}, v_{-+--+--+-}, \\
& v_{--+--+--+}, v_{++-++-++-}, v_{+-++-++-+}, v_{-++-++-++}\} \\
\cup\ & CP(v_{+++++-000}) \cup CP(v_{-++++-000}) \cup CP(v_{-+++--000}) \cup CP(v_{-++---000}) \\
\cup\ & CP(v_{-++0-+-00}) \cup CP(v_{-++-+-000}) \cup CP(v_{--+-0-0-0}).
\end{aligned} \tag{11}
$$

We have studied violations of these inequalities, assuming a symmetry between observers. We took as local observables $\mathbf{A}_1^i = \cos\phi_i\sigma_z + \sin\phi_i\sigma_x$, and $\mathbf{A}_2^i = \cos\phi_i\sigma_z - \sin\phi_i\sigma_x$ (for an explanation of such a choice, see further), with all $\phi_i = \phi$. The maximal eigenvalue of for five-qubit Bell operators related to $Z(5,1)$ was found to be 1.97435 with $\phi_i = \frac{\pi}{4}$. For $Z(7,1)$, $Z(9,1)$ and $Z(9,3)$ we assumed $\phi_i \equiv \phi$ and the maximal observed violation ratios were 1.84331, 2.18414 and 1.79497, respectively. One of the five-qubit states providing the maximal violation of the wrapping inequality with moduli defined by $Z(5,1)$, for Pauli observables constrained to the XZ plane, however without the $\phi_i = \phi$ constraint, reads

$$
\begin{aligned}
|\psi\rangle \approx\ & a|00000\rangle + b(-|00011\rangle + |00110\rangle + |01100\rangle - |11000\rangle - |10001\rangle) \\
& + c(|00101\rangle - |01010\rangle + |10100\rangle - |01001\rangle - |10010\rangle) + d(|00111\rangle + |01110\rangle + |11100\rangle - |11001\rangle - |10011\rangle) \\
& + e(|01011\rangle - |10110\rangle - |01101\rangle + |11010\rangle - |10101\rangle) + f(|01111\rangle + |10111\rangle - |11011\rangle + |11101\rangle + |11110\rangle) \\
& + g|11111\rangle,
\end{aligned} \tag{12}
$$

where $a = 0.462854, b = 0.161096, c = 0.19409, d = 0.181191, e = 0.220891, f = 0.107669$, and $g = 0.039699$. The state is given in the simplest form, a generalized Schmidt decomposition, defined in [22].

We can apply to this state a NOT map, which flips the Bloch vector of a qubit state ($\sigma_a \stackrel{NOT}{\to} -\sigma_a, a = x, y, z$), to all qubits. Such a map is composed of unitary rotations and a *full* transposition, and thus it is positive one. If we take an equal mixture $\rho = \frac{1}{2}(|\psi\rangle\langle\psi| + NOT^{\otimes 5}(|\psi\rangle\langle\psi|))$, the corresponding mean value reads 1.798, still much above 1. Notice that ρ possesses the same 4-party correlations as $|\psi\rangle\langle\psi|$, but it is globally uncorrelated. Hence it could not be used to violate any non-trivial WWWŻB inequality for five qubits .

Let us now briefly present a method to establish a necessary condition to violate the inequalities. We shall use the approach of Ref. [9]. In the course of constructing the inequalities the pairs of local observables occur either as $\frac{1}{2}(\mathbf{A_1^i} + \mathbf{A_2^i})$ or $\frac{1}{2}(\mathbf{A_1^i} - \mathbf{A_2^i})$. Take any possible pair of observables for the party i, that is $\mathbf{A}_1^i = \vec{n}_1^i \cdot \vec{\sigma}$ and $\mathbf{A}_2^i = \vec{n}_2^i \cdot \vec{\sigma}$, where $\vec{n}_k^i$ are unit vectors, and $\vec{\sigma}$ is a quasi-vector formed of Pauli operators for i. As $\vec{n}_1^i + \vec{n}_2^i$ and $\vec{n}_1^i - \vec{n}_2^i$ form an orthogonal pair, after some algebra we see that we can put $\frac{1}{2}(\mathbf{A}_1^i + \mathbf{A}_2^i) = \cos\phi_i \vec{w}^i \cdot \vec{\sigma}$ and $\frac{1}{2}(\mathbf{A}_1^i - \mathbf{A}_2^i) = \sin\phi_i \vec{w}^{\perp i} \cdot \vec{\sigma}$, where ϕ_i is certain angle, dependent on $\vec{n}_1^i$ and $\vec{n}_2^i$, and $\vec{w}^i$ and $\vec{w}^{\perp i}$ form an orthonormal pair. For simplicity we put $\vec{w}^i = \vec{x}^i$ and $\vec{w}^{\perp i} = \vec{z}^i$. We now can rewrite our Bell expression (for technical details of the approach consult [9]) for the wrapping inequality with moduli as a scalar product between two vectors, one containing trigonometric functions, and the other moduli of elements of the correlation tensor, $|T_{ij\ldots}| = |Tr\rho(\sigma_i \otimes \sigma_j \otimes \ldots)|$, with the indices extending from 0 to 3, and $\sigma_0 = \mathbf{I}$. The inequality 5 for three qubits

(which as aforementioned cannot be violated) gives

$$(|\cos\theta_1\cos\theta_2\cos\theta_3|, |\sin\theta_1\sin\theta_2\sin\theta_3|,\\ |\cos\theta_1\sin\theta_2|, |\cos\theta_2\sin\theta_3|, |\sin\theta_1\cos\theta_3|)\\ \cdot(|T_{333}|, |T_{111}|, |T_{310}|, |T_{031}|, |T_{103}|) \leq 1. \quad (13)$$

It is now easy to see that the norm of the first vector is at most 1 (the equality holds for the complete $Z(N,k)$). Thus, Cauchy-Schwartz inequality gives

$$T^2_{333} + T^2_{111} + T^2_{310} + T^2_{031} + T^2_{103} \leq 1, \quad (14)$$

as a sufficient condition to satisfy the inequality. If it holds for a given three-qubit state for all local bases, no inequality generated by this Z can be violated by this state.

Similarly, the sufficient condition for a state to satisfy all inequalities generated by ,e.g., $Z(5,1)$ is given by

$$T^2_{33333} + T^2_{11111} + T^2_{33310} + T^2_{03331} + T^2_{10333} + T^2_{31033}\\ + \; T^2_{33103} + T^2_{13310} + T^2_{01331} + T^2_{10133} + T^2_{31013} + T^2_{33101}\\ + \; T^2_{13110} + T^2_{01311} + T^2_{10131} + T^2_{11013} + T^2_{31101} \leq 1. (15)$$

The presented method leads to tight mutlipartite multisetting Bell inequalities which are an extension of the WWWŻB ones to multi order correlations. The actual construction was inspired by Ref. [15]. Our results certainly opens many interesting questions requiring further investigation. For example, how many different Z's can be constructed for given N, or what other symmetries can be imposed? A different task is construction with this method of inequalities violated by measurement data for experimentally accessible states. The degree of relevance of these inequalities in comparison to the known ones is another class of problems worth investigating.

While preparing this work we have learned about a new class of multisetting Bell inequalities for three qubits [21]. It is certainly interesting how these inequalities generalize to more qubits and if the two families, the one emerging from Vertesi-Pal inequalities and the one derived from here, converge.

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